AN ANALYTICAL AND NUMERICAL INVESTIGATION OF THE DISSIPATIVE CHAOS IN SEMICONDUCTOR SUPERLATTICES

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Abstract

The transport of electrons in a semiconductor superlattice miniband under the influence of electrical and magnetic fields, which are applied in different directions on the superlattice, is investigated. The time series diagrams and the Lyapunov exponent are computed using the fourth-order Runge-Kutta method. The numerical computations show that for particular values of the parameters, which depend on the superlattice characteristics and the fields applied on them, electrons show chaotic behaviors. In addition, for some other parameter values these behaviors become regular and non-chaotic. The presence of a magnetic field, perpendicular to the electrical field, is shown to reduce the chaotic areas in the motion of the electron. An alteration in electron average energy and velocity is attributed to application of the external fields, carrier scattering from other carriers, and the phonons’ and lattices’ faults. The study has important applications in computational physics and semiconductor chaotic simulations.

Keywords: Dissipative chaos; Semiconductor superlattices; Miniband; Electrons transport; Lyapunov exponent; fourth-order Runge-Kutta method

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1. INTRODUCTION

Semiconductor superlattices are formed from the consecutive layering of two kinds of semiconductors with different energy gaps on an under-layer which is usually of the same material as one of the semiconductors itself. They are formed using the molecular beam fitting method (MBE). The most prevalent of these superlattices are Al$_x$Ga$_{1-x}$As/GaAs, in which the combining semiconductor GaAs (with an energy gap of about 1.5eV) is placed, in the form of the every other layer, between the semiconductor layers of Al$_x$Ga$_{1-x}$As forming a ternary alloy. It is worth noting that for Al$_x$Ga$_{1-x}$As, $x<0.4$ and it has a direct band gap and its energy gap is just less than 2eV.

Owing to the fact that in forming superlattices very thin barriers are used [1], electron wave functions fall on each other from the surrounding wells and the tunneling of particles from one well to another by moving through the barrier becomes probable. This causes the quantumized energy levels degeneracy to be broken in each well. For the $N$ levels of each well, each degeneracy level is divided into $N$ additional levels. As a result, a band with $2N$ forms (considering the spin) is formed. The superlattice energy bands, which are formed like this, are called minibands.

Electron transport in semiconductor superlattices, due to their remarkable characteristics, has stimulated considerable interest in the engineering physics and electronic materials fabrication research community for some years. Important early studies have been communicated by Esaki and Tsu [2] and Price [3], both studies being developed at the IBM Thomas J. Watson Research Center, Yorktown Heights, New York. Esaki and Chang [4] later studied electronic transport properties in a GaAs-AlAs periodic structure which they termed the "superlattice" prepared by a molecular-beam epitaxy. They showed that the differential conductance in the superlattice direction initially decreases slowly, with a subsequent dramatic plummet to negative values. This stage is then ensued with an oscillatory behavior with respect to applied voltages under high strength fields. They elucidated these observations as the formation and expansion of a high-field domain, further establishing that the voltage period of the oscillation supplies the requisite energy of the first-excited band, a phenomenon which correlates strongly with that predicted by theory.

Palmier and Chomette [5] studied the transport properties at low applied electric field in a one-dimensional semiconductor superlattice using two scattering processes, namely the
deformation potential and the polar optical modes. They applied their model to GaAs/GaAlAs superlattice systems, calculating the mobility tensor as a function of the superlattice period.

Movaghar [6] developed a theoretical model transport of charge carriers in semiconductor multiple-quantum-well structures for motion perpendicular to the layers, based on strong electric fields with a feature enabling the possibility of describing the low-field ohmic region by using a simple scaling argument. He showed that disorder may significantly modify the nature of the physical effects predicted on the basis of the 'pure' eigenstates, with reference to experiments on magneto-transport perpendicular to the layers.

Further interesting experimental studies have been communicated by Peterson et al [7] and Noguchi et al [8]. Rossi et al [9] presented a seminal study of ultrafast dynamics of photoexcited carriers in semiconductor superlattices using a Monte Carlo solution of the coupled Boltzmann transport equations for electrons and holes. They were able in this work to describe kinetically the pertinent interaction mechanisms including intra-miniband and interminiband carrier-phonon scattering processes. They identified the significant influence of carrier–polar-optical-phonon interaction in determining the nature of the carrier dynamics in the low-density limit, and further showed that the miniband width, in comparison with the phonon energy, is the fundamental parameter for predicting the existence of Bloch oscillations.

Alekseev et al [10] presented a robust model for the dynamics of ballistic electrons in a miniband of a semiconductor superlattice (SSL) under the influence of an external, time-periodic electric field, using the semiclassical, balance-equation approach, incorporating elastic and inelastic scattering (as dissipation) and the self-consistent field generated by the electron motion. They computed for a range of values of the dissipation parameters the regions in the amplitude-frequency plane of the external field in which chaos arises, proposing that for terahertz external fields of the amplitudes achieved by industrial free-electron lasers, chaos may be observable in SSL systems. Scamarcio et al [11] reported observations of infrared emission between superlattice conduction minibands, for electrons resonantly injected into the first excited state miniband by an applied electric field make a radiative transition to the ground state miniband. They described experiments in AlInAs/GaInAs superlattices, achieving peak luminescence wavelengths of λ= 5 and 7 μm.

Alekseev et al [12] further studied the motion of ballistic electrons in a single miniband of a semiconductor superlattice driven by a terahertz laser polarized along the growth direction, investigating zones of complex dynamics, which may include chaotic behavior and symmetry-breaking. They computed the magnitude of the DC current and voltage that
spontaneously appears in regions of broken-symmetry for parameters characteristic of real semiconductor superlattices.

Gómez et al [13] examined the electron transmission probability in semiconductor superlattices in which barrier height is modulated by a Gaussian profile, showing that these structures serve as efficient energy band-pass filters and achieving a lower number of unintentional defects and enhanced performance. They further demonstrated that current-voltage characteristic presents negative differential resistance with peak-to-valley ratios which are significantly in excess of conventional semiconductor superlattices. Gmachl et al [14] quantified inter-sub-band optical absorption around 1.55 μm in GaN/AlGaN multiple quantum wells (MQWs) grown by molecular-beam epitaxy. They designed and fabricated QWs embedded in barriers consisting of a short period superlattice of narrow GaN QWs and only 65% AlN mole-fraction barriers, refining the electron Bragg confinement which allowed a peak absorption wavelengths as short as 1.52 μm. This approach also permitted modulation doping of the structures via doping of the narrow superlattice wells and subsequent charge transfer into the active well, which led to a decrease in the absorption line width, from ~200 to ~130 meV, for these structures.

One of the most detailed theoretical investigations of electric transport in semiconductor superlattices has been communicated by Wacker [15]. He has highlighted that such transport is dominated by pronounced negative differential conductivity and has lucidly reviewed the standard transport theories for superlattices, i.e. miniband conduction, Wannier-Stark-hopping, and sequential tunneling, elucidating their relation to each other via a comparison with a quantum transport model based on nonequilibrium Green functions.

Vashaee and Shakouri [16] developed a comprehensive theory for nonisothermal electron transport perpendicular to multilayer superlattice structures is presented, computing the current–voltage characteristics and the cooling power density using Fermi–Dirac statistics, density-of-states for a finite quantum well and the quantum mechanical reflection coefficient. They further compared their calculations with the experimental dark current characteristics of quantum well infrared photodetectors achieving excellent correlation over a wide temperature range for a variety of superlattice structures. This study superceded previous investigations in that lateral momentum conservation was satisfied for the case of electron transport in planar semiconductor barriers.

Perales et al [17] obtained comprehensive solutions of the drift-diffusion model equations for miniband transport in strongly coupled superlattices, basing their analysis on a single-miniband Boltzmann–Poisson transport equation with a BGK (Bhatnagar–Gross–Krook)
collision term founded on a consistent Chapman–Enskog expansion. The reduced drift-diffusion equation was solved numerically and travelling field domains and current oscillations presented. This work achieved a wide spectrum of frequencies and agreed well with experiments on GaAs/AlAs superlattices. An important outcome of the articles discussed is the identification of the probability of negative differential guidance in presence of the strong electrical fields. The negative differential guidance is formed because of the electron motion inside the miniband and somewhere in the momentum in which the electron effective mass is negative.

During recent years, semiconductor superlattices have been widely used in fabricating electronic devices [18-20]. Due to this, researchers have become interested in investigating the electron motion in superlattices and determining the locations of chaotic motion. Therefore, identifying the places in which electron behavior is chaotic is critical in the efficient and robust fabrication of modern semiconductor devices. Chaos can be formed in electron motion in semiconductor superlattices as a result of their non-isotropic effective mass and the application of external fields.

Articles written so far have, often, dealt with the electron behavior in superlattices under the influence of one external electrical field applied in the direction of the crystal growth [21-30]. We extend this approach here to analyze the transport of electrons in a semiconductor superlattice miniband under the influence of electrical and magnetic fields, which are applied in different directions on the superlattice. This work has important and immediate applications in semiconductor and solid state physics industries and has thus far not appeared in the engineering physics literature.

2. THEORETICAL FUNDAMENTALS

In this article, carrier transport is investigated in semiconductor superlattices in the presence of magnetic and electrical fields and in the form of the one-miniband model. Here, electron elastic scattering is considered from lattice impurities (the relaxation time approximation) and the scattering resulted from other electrons. In addition, the self-consistent electrical field caused by the electron's charge density is, also, considered. The external electrical field which is applied perpendicularly on the superlattice layers takes the form:

\[ E_{\text{ext}} = E_0 \cos \Omega t \hat{e}_z \]  

(1)
in which, \( E_y \) is the domain and \( \Omega_{E} \) is the electrical field frequency. In addition, the external magnetic field is defined as follows:

\[
B_{\text{ext}} = B_0 \cos \Omega_B t \left( \sin \theta \hat{e}_y + \cos \theta \hat{e}_z \right)
\]  

(2)

where \( B_0 \) is the domain, \( \Omega_B \) is the magnetic field frequency, and \( \theta \) is the angle which the magnetic field builds with \( z \)–axis (the superlattice layers growth direction). Owing to the fact that electrons move freely in layers plain (\( x, y \) plane), the energy relation for the electrons which belong to one miniband of a semiconductor superlattice can be written in the form [1, 21]:

\[
\varepsilon(P) = \frac{l}{2m^*} \left( p_x^2 + p_y^2 \right) + \frac{l}{2} \Delta \left[ 1 - \cos \left( \frac{p_z a}{\hbar} \right) \right]
\]  

(3)

in which the Tang-Been model is used for computing energy in the \( z \) direction, and \( P = \hbar k \) is the crystal momentum, \( m^* = 0.07m_e \) is the effective mass of an electron moving in the \( x \) and \( y \) directions, \( m_e \) is the mass of the free electron, \( a \) is the superlattice alternating, and \( \Delta \) is the miniband width. Using the Lorentz force equation, one can derive the energy time changes and the electron velocity. Also, the self-consistent field time changes can be defined using Ampere’s electromagnetic law. Finally, the normalized equations of the electron motion in one miniband of a superlattice may be presented, thus:

\[
\dot{w} = -E v_z - \gamma_{\varepsilon} \left( w - w_0 \right)
\]  

(4)

\[
\dot{v}_x = \frac{-eB_0 \cos \Omega_B t}{m^*} \left( v_y \cos \theta - v_z \sin \theta \right) - \gamma_{\varepsilon} v_x
\]  

(5)

\[
\dot{v}_y = \frac{eB_0 \cos \Omega_B t}{m^*} v_x \cos \theta - \gamma_{\varepsilon} v_y
\]  

(6)

\[
\dot{v}_z = - \left( E + \frac{e\Delta a^2}{2\hbar^2} v_x B_0 \cos \Omega_B \sin \theta \right) \left( w + \frac{\Delta a^2 m^*}{4\hbar^2} \left( v_x^2 + v_y^2 \right) \right) - \gamma_{\varepsilon} v_z
\]  

(7)
\[ \dot{E} = v_z - \alpha E + \alpha \omega_s \cos \Omega_E t - \omega_s \Omega_E \sin \Omega_E t \]

in which \( \gamma \) is the energy relaxation frequency, \( \gamma_v \) is the electron velocity relaxation frequency in the direction of \( i = (x, y, z) \), \( \alpha \) is the self-consistent field relaxation frequency, and \( \omega_s = \frac{e a E_0}{\hbar} \), which is famously known as the Stark frequency. Also, \( w \) and \( w_0 \) are the electron energy and the equilibrium energy, respectively, which, the latter, is defined as:

\[ w_0 = \frac{k_B T}{4 \Delta} \left( I_1 \left( \frac{\Delta}{2 k_B T} \right) \right) \]

\[ I_0 \left( \frac{\Delta}{2 k_B T} \right) \]

in which \( I_1 \) and \( I_0 \) are the zero and first-order reformed Bessel functions, \( T \) is the lattice temperature, and \( k_B \) is the Boltzmann constant. In Eqs. (4) and (8), all relaxation coefficients are normalized by dividing into \( \omega_E \equiv \left[ \frac{2 \pi e^2 n a^2 \Delta}{\hbar^2} \right]^{1/2} \). These equations show that a change in the electrons average energy and velocity is attributable to the application of the external fields, carrier scattering from other carriers, and the phonon and lattice faults. It is worth noting that the external fields are of insufficient strength to be able to modify the band structure; therefore, in all of the above-mentioned computations, the dispersion Eq. (3) is used.

3. NUMERICAL RESULTS

In the numerical computations, we integrate from motion Eqs. (4) and (8) using the highly efficient, well-tested fourth-order Runge-Kutta method with optimized steps and then draw the velocity time series diagram and the largest Lyapunov exponent. This numerical technique has been used successfully in numerous complex multiphysical problems. Takhar et al [31] studied hydromagnetic heat transfer in porous media using Runge-Kutta methods. Bég et al [32] investigated vorticity diffusion effects in permeable convection with Runge Kutta and difference schemes. Further studies in thermophysical and geophysical fluid
dynamics employing variations of Runge-Kutta quadrature have been communicated by Chamkha et al [33] and Takhar et al [34]. In the context of solid state physics, an excellent summary of Runge-Kutta methods has been given by Parechi and Russo [35]. Kuo et al [36] used a fourth order R-K method to simulate passive Q-switching with solid-state saturable absorbers in laser engineering. Prodan and Nordlander used Runge-Kutta numerical quadrature to study electronic structure of metallic nanoshells for a large nanoshell (of 10 nm in diameter) containing more than 2.5×10⁴ conduction electrons. Becker et al [38] employed R-K methods to investigate the Cameron bands luminescence from a new thermoluminescence (TL) system (CO-doped solid Ar) - with selective excitation by synchrotron radiation and thermally stimulated luminescence methods. They showed that activation energies and the frequency factors of charge traps are very efficiently computed by numerical fitting of the TL emission spectrum with a Runge-Kutta solver. Maiorov et al [39] studied kinetics of plasma particles around a stationary dust grain in the presence of an ion flow is studied using a three-dimensional molecular dynamics simulation method, based on a Runge-Kutta numerical solver. Edwards and Burnett [40] developed a highly efficient Runge-Kutta method for solving the transient nonlinear Schrödinger equation with an external potential, achieving excellent accuracy for the case of a dilute Bose-condensed assembly of trapped neutral atoms where the potential varies on the same scale as the condensate. Cranfield et al [41] have also successfully used Runge-Kutta numerical techniques in perturbation analysis of oscillating magnetic fields in the radical pair mechanism. In the present study we shall consider superlattice parameters like [42]:

\[ a = 10^{-8} \, m, n = 3 \times 10^{21} \, m^{-3}, \Delta = 35.2 \times 10^{-21} \, J \] (10)

In experiments done in the references [43, 44], the areas of the damping parameters are determined as \[ 0 \leq \frac{\alpha}{\omega_E} \leq 0.2, \quad \frac{\gamma_e}{\omega_E} \leq 0.2, \quad \text{and} \quad 0 \leq \frac{\gamma_v}{\omega_E}. \] First, we investigate the electron behavior in the presence of the electrical field and in the absence of the magnetic field. Fig. (1) shows velocity time series for values \[ \frac{\omega_j}{\omega_E} = 1.6, \quad \frac{\Omega_k}{\omega_E} = 0.2, \quad \frac{\alpha}{\omega_E} = 0.01, \quad \frac{\gamma_v}{\omega_E} = \frac{\gamma_e}{\omega_E} = 0.02, \]
and \[ \frac{\gamma_v}{\omega_E} = \frac{\gamma_e}{\omega_E} = 0. \] Also, this figure shows electron velocity for these chaotic parameters values because the velocity changes in a non-normalized (non-periodic) way. One can be
sure of the presence of chaos by determining the Lyapunov exponent. By computing the
distance between two routes, with very close initial conditions, one can compute the
Lyapunov exponent. If the divergence rate of the two routes is in the form of an exponent
function with a positive exponent, the motion is chaotic. Consider two close routes with a
distance $d_{n}$, we compute $d_{n}$, distance between these routes numerically, and in cases
where $\frac{d_{n}}{d_{0}}$ is larger than a value between 2 to 3, we have, again, normalize $d_{n}$ to $d_{0}$. The
Lyapunov exponent is defined in this form:

$$\sigma = \lim_{\tau \to \infty} \sigma(\tau)$$

(11)

in which:

$$\sigma(\tau) = \frac{1}{\tau} \sum_{n=1}^{\infty} \ln \left( \frac{d_{n}}{d_{0}} \right)$$

(12)

$\tau \equiv \sum_{n=1}^{\infty} \Delta \tau_{n}$ is the normalized time and $\Delta \tau_{n}$ is the (normalized) time distance between the $n$th step and $(n-1)$th step. Also, time is normalized to $\frac{1}{\omega_{e}}$. In numerical computations with
infinite time, this is interpreted to mean reaching a time in which the diagram $\sigma(\tau)$ reaches a
rather continuous tend on the basis of the time.

For the sake of an improved investigation of the kind of motion, $\sigma(\tau)$ is drawn in Fig. (2)
for Fig. (1) parameters values. Also, in the numerical solution of the equations and in
drawing $\sigma(\tau)$, the initial conditions are:

$$\begin{align*}
  w_{1}(0) &= w_{2}(0) = -1 \\
  v_{x_{1}}(0) &= v_{x_{2}}(0) = 0 \\
  v_{y_{1}}(0) &= v_{y_{2}}(0) = 0 \\
  v_{z_{1}}(0) &= v_{z_{2}}(0) = 0 \\
  E_{1}(0) &= \omega_{e} \quad , E_{2}(0) = E_{1}(0) + \delta_{0}
\end{align*}$$

(13)
Here, $\delta$ is a small value of magnitude 0.001 and, also, indexes 1 and 2 show the routes 1 and 2, respectively. As can be seen in Fig. (2), $\sigma(\tau)$ shows a constant value as big as 0.03 for large times ($\tau > 10000$). Therefore, by a good approximation, one can say that the Lyapunov exponent $\sigma$ is positive. The Lyapunov exponent’s positivity highlights the presence of chaos in the absence of a magnetic field.

One can observe that in the absence of the magnetic field the electron motion for the utilized parameters in Figs. (1) and (2) is chaotic. Now, we change the parameters as:

\[
\frac{\Omega_E}{\omega_E} = 1 \\
\frac{\omega_1}{\omega_E} = 0.1 \\
\frac{\gamma_v}{\omega_E} = \frac{\gamma_v}{\omega_E} = 0 \\
\frac{\alpha}{\omega_E} = 0.01 \\
\frac{\gamma_v}{\omega_E} = \frac{\gamma_v}{\omega_E} = 0.01
\]

(14) (15) (16) (17) (18)

Fig. (3) shows the electron mean velocity time series for the new parameters. As can be seen from this figure, velocity changes periodically and there is no sign of chaos. In this diagram, the fundamental frequency is the external field frequency with a greater period which is on it, due to the non-linear equations.

Fig. (4) shows $\sigma(\tau)$ for new parameters. As can be seen from this figure, $\sigma(\tau)$ shows a negative constant value for large times. Therefore, the Lyapunov exponent is negative, which shows a regular motion (non-chaotic) for these new parameters. The above-mentioned results are equivalent to the reference results [43] which have investigated the chaos in superlattices just in presence of one electrical field. This emphasizes the accuracy of the Eqs. (4) to (8).
In this section, we deal with the numerical solution of the motion equations in the presence of a magnetic field with a domain of \( \frac{B_0}{\omega_E} = 10^{-12} T_s \) and a frequency of \( \frac{\Omega_B}{\omega_E} = 0.1 \). In order to solve the equations numerically, we consider the same parameters values used in Fig. (1) (i.e., the values which cause the electron’s motion to be chaotic). We aim to investigate how applying a magnetic field affects the chaotic motion of the electron. We apply the magnetic field perpendicularly on the electrical field and in the direction of \( y \) on the superlattice. Figs. (5) and (6) show the electron velocity time series and \( \sigma(t) \) in the presence of the magnetic field, respectively.

As can be seen in these figures, the electron velocity is periodic for the large enough time and, also, \( \sigma(t) \) shows the constant and negative value for the large time which represents the negative Lyapunov exponent. Therefore, chaos, in presence of the magnetic field, fades. It is worth noting that when we consider the magnetic field parallel to the electrical field, the electron motion remains chaotic, which can be seen in Figs. 1 and 2.

For the sake of reaching a better view of the qualitative behavior of the system, we draw the phase space diagram for the electrical field domain on the basis of its frequency. We do this by changing the electrical field domain and frequency by a 0.01 step and, then, compute the Lyapunov exponent for them. Figs. (7) and (8) are drawn for relaxation parameters \( \frac{\gamma_v}{\omega_E} = \frac{\gamma_{v_1}}{\omega_E} = 0 \), \( \frac{\gamma_v}{\omega_E} = \frac{\gamma_{v_1}}{\omega_E} = 0.1 \), and \( \frac{\alpha}{\omega_E} = 0.01 \). The dark areas show the chaotic areas and the white ones deal with the periodic motion. Fig. (7) is drawn in the absence of the magnetic field and Fig. (8) is considered in the presence of the magnetic field.. It is observed that the magnetic field application \( \left( \frac{B_0}{\omega_E} = 10^{-12} T_s \right. \) and \( \left. \frac{\Omega_B}{\omega_E} = 0.1 \right) \), which is as large as the electrical field, causes the chaos to be eliminated on the left side of the diagram.

4. CONCLUSIONS

In this article, we have shown that in presence of the electrical field and in the absence of the magnetic field, the electron motion in the superlattices, for some parameters related to the superlattice and the external field, can be chaotic. In addition, for some other parameters, it can be periodic or non-chaotic. However, in presence of the orthogonal
electrical and magnetic fields, chaos can be eliminated and the electron motion in superlattices, for some parts of the phase space, becomes regular or non-chaotic.

5. REFERENCES


6. FIGURE CAPTIONS

Figure 1. The electron velocity in the absence of the magnetic field on the basis of time for parameters $\frac{\gamma_v}{\omega_E} = \frac{\gamma_e}{\omega_E} = 0.02$, $\frac{\Omega_E}{\omega_E} = 0.2$, $\frac{\omega_i}{\omega_E} = 1.6$, $\frac{\gamma_v}{\omega_E} = \frac{\gamma_e}{\omega_E} = 0$, and $\frac{\alpha}{\omega_E} = 0.01$. The electron motion does not have a particular period and is chaotic.

Figure 2. Diagram $\sigma(\tau)$ in the absence of the magnetic field for the parameters of Fig. (1). It can be seen that $\sigma(\tau)$ shows a constant and positive value of 0.03 for $\tau > 10000$, which puts a true mark on the presence of chaos.

Figure 3. The electron velocity time series in the absence of the magnetic field for the parameters $\frac{\omega_i}{\omega_E} = 0.1$, $\frac{\Omega_E}{\omega_E} = 1$, $\frac{\gamma_v}{\omega_E} = \frac{\gamma_e}{\omega_E} = 0$, $\frac{\gamma_v}{\omega_E} = \frac{\gamma_e}{\omega_E} = 0.01$, and $\frac{\alpha}{\omega_E} = 0.001$. The electron motion is periodic (non-chaotic).

Figure 4. Diagram $\sigma(\tau)$ in the absence of the magnetic field for the parameters of Fig. (3). The Lyapunov exponent’s being negative shows the regular and non-chaotic motion.

Figure 5. The electron velocity time series in the direction of $z$, in presence of the magnetic field perpendicular on the electrical field, and on the basis of the normalized time for the parameters of Fig. (1). In presence of the magnetic field, the electron motion becomes regular and non-chaotic.

Figure 6. Diagram $\sigma(\tau)$ in presence of the magnetic field, perpendicular on the electrical field. The presence of the magnetic field causes the annihilation of chaos.

Figure 7. Phase space diagram in the absence of the magnetic field. The dark areas show the chaotic motion and the white areas represent the periodic (non-chaotic) motion.

Figure 8. Phase space diagram in presence of the magnetic field. The dark areas show the chaotic ones and the white areas represent the periodic (non-chaotic) motion.
7. FIGURES

Figure 1. The electron velocity in the absence of the magnetic field on the basis of time for parameters $\frac{\gamma_r}{\omega_E} = 0.02$, $\frac{\gamma_s}{\omega_E} = 0.2$, $\frac{\omega_s}{\omega_E} = 0.2$, $\frac{\gamma_s}{\omega_E} = 1.6$, $\frac{\gamma_s}{\omega_E} = 0$, and $\frac{\alpha}{\omega_E} = 0.01$. The electron motion does not have a particular period and is chaotic.

Figure 2. Diagram $\sigma(\tau)$ in the absence of the magnetic field for the parameters of Fig. (1). It can be seen that $\sigma(\tau)$ shows a constant and positive value of 0.03 for $\tau > 10000$, which puts a true mark on the presence of chaos.
Figure 3. The electron velocity time series in the absence of the magnetic field for the parameters $\frac{\omega}{\omega_E} = 0.1$, $\frac{\Omega}{\omega_E} = 1$, $\gamma_{v_r} = \gamma_{v_e} = 0$, $\gamma_{v_r} = \gamma_{v_e} = 0.01$, and $\frac{\alpha}{\omega_E} = 0.001$. The electron motion is periodic (non-chaotic).
Figure 4. Diagram $\sigma(\tau)$ in the absence of the magnetic field for the parameters of Fig. (3). The Lyapunov exponent’s being negative shows the regular and non-chaotic motion.

Figure 5. The electron velocity time series in the direction of z, in presence of the magnetic field perpendicular on the electrical field, and on the basis of the normalized time for the parameters of Fig. (1). In presence of the magnetic field, the electron motion becomes regular and non-chaotic.
**Figure 6.** Diagram $\sigma(\tau)$ in presence of the magnetic field, perpendicular on the electrical field (the presence of the magnetic field causes the annihilation of chaos).

**Figure 7.** Phase space diagram in the absence of the magnetic field. The dark areas show the chaotic motion and the white areas represent the periodic (non-chaotic) motion.
Figure 8. Phase space diagram in presence of the magnetic field. The dark areas show the chaotic ones and the white areas represent the periodic (non-chaotic) motion.