Understanding Chaos using Discrete-Time Map for Buck Converter
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Abstract-----Research in nonlinear dynamics and complexity has made remarkable progress in recent years. Almost all power electronic circuits exhibit some kind of nonlinear behavior e.g., quasi-periodicity, sub-harmonic oscillations, bifurcation, chaos. The aim of this paper is to investigate the nonlinear phenomenon and chaotic behavior in a DC-DC buck converter. The derivation of the discrete-time map for the buck converter is given. This map is simulated and the results infer chaotic behavior.

Index terms: - Bifurcation, chaos, buck converter, strange attractor.

I. INTRODUCTION

A. DC-DC Buck Converter

A DC-DC converter also known as chopper or switching regulator is used to obtain a variable DC voltage from a constant voltage DC source as shown in fig. 1.

Fig. 1: Basic DC-DC chopper

The DC-DC converters are widely used in regulated switched mode power supplies (SMPS) and in DC motor drive applications. There are two fundamental kinds of DC-DC converters: The step-down or buck converter and the step-up or boost converter. The buck converter produces an output voltage that is less than or equal to the input voltage whereas the boost converter provides an output voltage that is greater than or equal to the input voltage. However, it must be kept in mind that a step-down voltage converter is also a step-up current converter and vice versa because the input power must equal the output power [1]. Fig. 2 below shows a DC-DC buck converter.

Fig. 2. Basic step-down converter

It consists of DC input voltage $V_S$, controlled switch S (in form a transistor Q), diode D, filter inductor L, filter capacitor C and load resistor R.

B. What is chaos?

Chaos is an aperiodic behavior in a deterministic system that shows sensitive dependence on initial conditions. The mathematical definition of chaos is unpredictable long time behavior arising in a deterministic dynamic system because of sensitivity to initial conditions (termed the butterfly effect) [2]-[4].

Actually, these systems are fundamentally deterministic i.e., the precise knowledge of initial conditions of the system allows prediction of future behavior of that system. Hence the term chaos is used to explain the apparently complex behavior of so called simple, linear and well behaved systems [2]-[4].

Chaos theory is defined as the qualitative study of unstable aperiodic behavior in deterministic nonlinear dynamic systems. Thus chaos may be described as a bounded, aperiodic, and noisy like oscillation: a deterministic system appears to behave randomly even though there is no random input. Chaotic system exhibits apparently random and unpredictable behavior [2]-[4].
C. Chaos in DC-DC Converters

Although Van der Pol in 1927 first observed nonlinear dynamics in electronic circuits, it is only recently that the existence of bifurcations and chaos in DC-DC converters has been proposed [5]-[7].

DC-DC converters have always been designed to operate in only one type of periodic operation, termed period-1 operation in which all the waveforms repeat at the same rate as the driving clock pulse. Thus converters are expected to work stably in this regime under all possible disturbances. But period-1 operation is not the only possibility. For example, under certain conditions, the circuit may operate in a period-n regime in which the periods of all waveforms are exactly n times that of the driving clock pulse [5]-[7].

The complexity in the operation of DC-DC converters can be appreciated, where a variety of operational regimes exist and a large number of parameters may affect the stability of a particular regime. As parameters vary, the operation can go from one regime to another, sometimes in an abrupt manner. Such a phenomenon, where one regime fails to operate and another picks up, is termed bifurcation [5]-[7].

Thus even when a converter works satisfactorily in a one regime, it could fail to operate as expected if some parameters are changed, causing it to assume another regime. If the newly assumed regime is an undesirable one, locating the bifurcation boundary becomes imperative. The period-1 stable operation is the preferred operation for most industrial applications; it represents only one particular operating regime. Because of the existence of many possible operating regimes, it would be of practical importance to have a thorough understanding of what determines the behavior of the circuit to ensure a desired operation or to avoid an undesirable one [5]-[7].

II. DISCRETE TIME MAP FOR BUCK CONVERTER

The derivation of discrete-time map for buck converter given below is taken from a work of S. K. Tse [5-6]. A discrete-time map is a dynamic system which works in increments. It takes the conditions at some time t and gives the conditions at a later time, t+a e.g., the logistic map that is a population growth model. Such maps are also called iterative maps in which the value of x at t=nT is expressed in terms of that at t = (n-1)T, where T is the period of the switching cycle. As is well known in the literature of chaos, such maps facilitate the investigation of chaos and sub-harmonics.

In terms of the circuit operation, a DC-DC converter can be described as a piece-wise switched electrical circuit whose topology toggles between a number of linear circuits in some predefined manner. In simple DC-DC converters such as the buck, boost and buck-boost converters the involved circuits are second-order circuits containing an inductor, a capacitor, a switch (externally controlled switch) and a diode (internally controlled switch).

The simple buck, boost and buck-boost converters having two independent storage elements are second-order systems. Nevertheless, a close inspection of the inductor current waveform reveals that the inductor current is identically zero at the start of each switching period when operating in discontinuous conduction mode (DCM), i.e.,

\[ i_L(nT) = 0 \text{ for all integers } n \]  \hspace{1cm} (1)

Where ‘T’ is the switching period. Thus the inductor current is no longer a dynamic variable. As a result, the converter reduces to first-order, with the capacitor voltage serving as the only state variable. Therefore, the discrete-time model for a DC-DC converter operating in discontinuous conduction mode takes the form of a first-order iterative map:

\[ v_{n+1} = f(v_n) \]  \hspace{1cm} (2)

Where subscript ‘n’ denotes the value at nth switching instant (t=nT) and other symbols have their usual meanings. Therefore, stacking consecutive solutions over a switching period would result the iterative map which expresses x(t_{n+1}) in terms of x(t_n). The resulting first-order iterative map describes the open-loop dynamics of the system. Then with the loop closed, the feedback factor becomes a parameter that can be varied at will. The resulting discrete-time map is of the form

\[ v_{n+1} = f(v_n, d_n) \]  \hspace{1cm} (3)

Where ‘d_n’ denoted the feedback factor and the duty cycle during the nth period given as:

\[ d_n = \frac{t}{T} \]  \hspace{1cm} (4)
Looking at equation (3), it is clear that \( d_n \) is an input variable that can be varied to regulate the system. In practice, the value of \( d_n \) is adjusted via a pulse-width modulator that turns on switch S at \( t=nT \) for a duration \( d_nT \). When \( d_n \) is assigned to fixed value, the system is said to be uncontrolled or open-loop. On the other hand, when \( d_n \) is varied according to input, the system is termed as closed-loop system.

Simplifying algebraic equations the discrete-time map for buck converter can be given as:

\[
v_{n+1} = \alpha v_n + \frac{\beta (d_n) V_{in} (V_n - v_n)}{v_n}
\]

Where

\[
v_n = v_c(t_n) = v_c(t_{n+1})
\]

\[
\alpha = 1 - \frac{T}{CR} - \frac{T^2}{2C^2R^2}
\]

\[
\beta = \frac{T^2}{2LC}
\]

The map represented by (5) generates values of \( v_n \) at \( t=nT \) for all positive integers \( n \). Obviously the usefulness of this map lies in its ability to predict the flow of the system. In particular, if the system has a contraction it is guaranteed it will converge to a unique fixed value. This discrepancy can be removed by focusing attention on the system controlled by the simple feedback scheme.

\[
\delta d_n = k \delta v_n
\]

Where \( k \) is the feedback factor and can be chosen to modify the closed-loop dynamics. In practice, DC-DC converters are controlled via feedback mechanism. The usual control objective is to keep the output voltage fixed. For simplicity, we consider a proportional feedback which effectively samples the output voltage and generates an error signal from which the duty cycle is derived, i.e.,

\[
d_n = H(D + k(v_n - V_{ref}))
\]

Where \( D \) is the steady-state duty cycle, \( k \) is the small-signal feedback gain, and \( H(.) \) accounts for the limited range of the duty cycle between 0 and 1:

\[
H(x) = \begin{cases} 
0 & x < 0 \\
1 & x > 1 \\
x & \text{otherwise}
\end{cases}
\]

Combining this control equation with the discrete-time map of the system, we yield a discrete-time map for the closed-loop system. For the buck converter, we get:

\[
v_{n+1} = \alpha v_n + \frac{\beta (H(D + k(v_n - V_{ref}))) V_n (V_n - v_n)}{v_n}
\]

Where uppercase letters denote steady state values of the variables concerned as usual and \( H(.) \) represents the inherent saturating nonlinearity of the pulse-width modulator.

An example will help visualize the situation. The simulation will use the following parameters: \( T=33.33\mu s \), \( V_{in}=33V \), \( V_c=25V \), \( C=222\mu F \), \( R=12.5\Omega \). This gives \( T/CR=0.12 \), \( TR/L= 20 \), \( D= 0.4717 \). Direct substitution gives

\[
v_{n+1} = 0.8872v_n + \frac{1.2 \times 33 \times (33 - v_n) \times H(d_n)^2}{v_n}
\]

Where

\[
d_n = 0.4717 - k(v_n - 25)
\]

The bifurcation diagrams and the time series analysis for the maps represented by the equations (5) and (12) have been plotted in the next section. They represent typical unimodal map, i.e., one that has one local minimum or maximum, which is well known to exhibit a period-doubling route to chaos. Bifurcation
diagram is essentially a summary chart of the different types of behavior exhibited by a system when some parameters are varied.

III. TIME SERIES OF BUCK CONVERTER

In the previous section, we put together the discrete-time map for buck converter given as:

\[ v_{n+1} = \alpha v_n + \beta (H(D + k(v_n - V_{ref}))) V_a (V_a - v_n) \]

Now we will produce successive iterated solutions, or orbits to the aforementioned map for different values of control parameter k (feedback factor). In this process, at each step in the iteration, the current value of \( v_n \) is fed back to produce the next value, then this value is used to produce the next and so on.

In fig. 3, for \( k=0.10 \), the first thirty iterated solutions to the buck converter map \( v_n \) are plotted against \( n \). This final solution is known as period-1 orbit, as the iterates tend to a fixed value where \( v_{n+1} = v_n \) for large \( n \).

For \( k=0.14 \) in fig. 4, the solutions rapidly converge to two alternating attracting fixed points. These solutions to the buck converter map repeat every second value, i.e., \( v_{n+2} = v_n \). This is termed as period-2 orbit.

In fig. 5, for \( k=0.16 \), the solutions to the buck converter map repeat every fourth value, i.e., \( v_{n+4} = v_n \). This is called period-4 orbit.

Fig. 6, for \( k=0.18 \), is shown. In it the iterated solutions do not converge and further iterations will never produce a repeating and hence periodic sequence of solutions. This aperiodic behaviour is known as chaotic orbit or simply chaos.
In fig. 7, for $k=0.22$, the behaviour is still *aperiodic*.

Fig. 8 for $k=0.26$ also exhibits *chaotic orbit*.

Fig. 9 is drawn for $k=0.28$. This final solution is again *period-1 orbit*, as the iterates tend to a fixed value where $v_{n+1} = v_n$ for large $n$.

### IV. BIFURCATION DIAGRAMS OF BUCK CONVERTER

Fig. 10(a) shows bifurcation diagram for buck converter equation without $d_o$ and fig. 10(b) depicts the bifurcation diagram for buck converter equation $d_o$. Fig. 10(b) also clearly shows period-3 and period-5.
V. CONCLUSIONS

In the field of power electronics, chaotic circuits are being examined both theoretically and experimentally. An analysis of pulse-width modulated (PWM) controlled DC-DC buck converter has been performed with the help of simulations. Besides time series, we have employed bifurcation diagram which is the most commonly used tool for capturing chaotic behavior.

The discrete-time map has provided a fast and easy way of obtaining the time series and bifurcation diagrams. We have seen that iterating a converter map, the iterated solutions finally settle down to a final behavior type either periodic or aperiodic. In addition, bifurcation diagrams confirm the mathematical results given by the time series of buck converter.

REFERENCES


