A Highly Chaotic Attractor for a Dual-Channel Single-Attractor, Private Communication System

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Abstract: A one-parameter highly chaotic attractor is presented and its application to a dual-channel, single-attractor, private communication system is demonstrated based on self-synchronization and chaotic masking techniques. Only a single attractor is required for a dual-channel transmitter or receiver, and can be either the well-known Lorenz attractor, the Lorenz-like attractor, or the one-parameter highly chaotic attractor developed in this paper. The latter is particularly well suited for an application to private communications due to the relatively high values of both the maximum Lyapunov exponent of 2.6148 and the maximum Kaplan-Yorke dimension of 2.1921. An advantage of the dual channel is the possibly twice increase in higher speed.

Keywords: Highly chaotic attractor, self-synchronization, dual-channel single-attractor private communications.

1. Introduction

Since the discovery of the eminent Lorenz chaotic attractor in 1963 [1], the study of chaotic behaviour in nonlinear systems has attracted great attention due to possible applications in various fields of science and technology. In particular, the pioneer work of Pecora and Carroll [2-4] based on chaos synchronization has attracted special interests in applications of chaos to various communications including private communication systems. A certain class of chaotic systems possesses a self-synchronization property [2-4], if it can be decomposed into at least two subsystems, i.e. a drive system (transmitter) and a stable response subsystem (receiver), that synchronize when coupled with a common drive signal. The ability of self-synchronization in the Lorenz systems and others for communications has been found experimentally to be highly robust to perturbations in the drive signal leading to the chaotic signal masking techniques [5,6]. Such techniques, however, have been applied mainly to a single channel of communication systems.

Most existing chaotic attractors with five terms [7-10], six terms [e.g. 11] or seven terms [e.g. 1] in three first-order ordinary differential equations (ODEs) have been suffered from relatively low values of either the Lyapunov exponent ($L$), or the Kaplan-Yorke dimension ($D_{KY}$). As the former ($L$) is a measure of chaoticity whilst the latter ($D_{KY}$) is a measure of complexity (or
strangeness) [8], high values of both are desirably expected for applications of chaos to private communication systems.

In this paper, a one-parameter highly chaotic attractor is presented and its application to a dual-channel, single-attractor, private communication system is demonstrated. Only a single attractor is required for a dual-channel transmitter or receiver, and can be either the well-known Lorenz attractor [1], the Lorenz-like attractor [11], or the one-parameter highly chaotic attractor developed in this paper. The latter is an improved version of [11] and is particularly well suited for an application to private communications as a result from the highest value of the Lyapunov exponent and the relatively high value of the Kaplan-Yorke dimension. As an example, robust self-synchronization of only the one-parameter highly chaotic attractor is demonstrated for a dual-channel single-attractor private communication system using the signal masking technique. An advantage of the dual channel is the possibly twice increase in higher speed.

2. A One-Parameter Highly Chaotic Attractor
An existing Lorenz-like chaotic attractor [11] has been described as a set of three first-order, autonomous, ODEs of the form:

\[
\begin{align*}
\dot{x} &= a(y - x), \\
\dot{y} &= bx - dxy, \\
\dot{z} &= xy - cz.
\end{align*}
\] (1)

where \(a = 5.8, b = 16, c = 1.8\). The parameter \(d\) simply scales the size of the attractor and therefore \(d\) can be any factor, e.g. \(d = b/c\). A trivial simplification is to set one or more of the four parameters to 1. One way to do this without destroying the chaos is to set the parameters \(d = c = 1\).

Therefore, the system (1) becomes a simpler ODE as follows:

\[
\begin{align*}
\dot{x} &= a(y - x), \\
\dot{y} &= bx - xz, \\
\dot{z} &= xy - z.
\end{align*}
\] (2)

In an attempt to adjust the parameters \(a\) and \(b\) for the maximum Lyapunov exponent, the parameter \(a\) in (2) increases to more than a hundred which is undesirable because the solution of state variables \(x, y, z\) in (2) occupies a wide dynamic range and the parameter \(b\) must also be adjustable to be a very large number in order to maintain its chaotic behaviour.

An alternative approach is to scale down the variables \(x, y\) and \(z\) to be smaller whilst a relatively high value of the maximum Lyapunov exponent can be maintained. For example, let \(a = 10, u = x/20, v = y/20\) and \(w = z/40\). Therefore, \(\dot{u} = 10(v - u), \dot{v} = bu - 40uw, \dot{w} = 10uv - w\). Consequently,

\[
\begin{align*}
\dot{x} &= 10(y - x), \\
\dot{y} &= bx - 40xz, \\
\dot{z} &= 10xy - z.
\end{align*}
\] (3)

It can be seen from (3) that \(b\) is now the only single controllable parameter, and therefore (3) is a one-parameter chaotic attractor. Figure 1(a) illustrates a bifurcation diagram exhibiting a period-doubling route to chaos of the peak of \(z\) of (3) versus \(b\) which is varied from 0 to 600. It is apparent that there are some periodic windows in the chaotic region. The chaotic behavior
A Highly Chaotic Attractor for a Dual-Channel Single-Attractor Private Communication System disappears quickly for \( b \leq 0 \). Figures 1(b) and (c) shows the corresponding Lyapunov exponent spectrum and the Kaplan-Yorke dimension \( D_{KY} \) of (3), respectively. It can be observed from Figs. 1 (a-c) that, at \( b \) near 296.5, the maximum Lyapunov exponent \( \lambda_{(\text{max})} = 2.6148 \) and the maximum Kaplan-Yorke dimension \( D_{KY(\text{max})} = 2.1921 \).

![Figure 1](image)

Table 1 summarizes the Lyapunov exponent \( (L) \) and the Kaplan-Yorke dimension \( (D_{KY}) \) of the system (3) developed in this paper compared to those of others. It is apparent from Table 1 that the value of \( L = 2.6148 \) of the system (3) is the highest, although the value of \( D_{KY} = 2.1921 \) of the system

<table>
<thead>
<tr>
<th>Attractors</th>
<th>Lyapunov Exponents L</th>
<th>( D_{KY} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCLS [8]</td>
<td>(0.2101, 0, -1.2101)</td>
<td>2.2354</td>
</tr>
<tr>
<td>Lorenz-Like [11]</td>
<td>(0.5907, 0, -3.907)</td>
<td>2.0701</td>
</tr>
<tr>
<td>Lorenz [12]</td>
<td>(0.9056, 0, -14.5723)</td>
<td>2.0621</td>
</tr>
<tr>
<td>Liu [12]</td>
<td>(1.2890, 0, -20.2890)</td>
<td>2.0635</td>
</tr>
<tr>
<td>Five Terms [7]</td>
<td>(1.4913, 0, -6.4939)</td>
<td>2.2301</td>
</tr>
<tr>
<td>Chen [12]</td>
<td>(2.0836, 0, -12.0836)</td>
<td>2.1724</td>
</tr>
<tr>
<td>This Paper</td>
<td>(2.6148, 0, -13.6148)</td>
<td>2.1921</td>
</tr>
</tbody>
</table>
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(3) is the third highest, which is only 2% smaller than that of the maximally complex Lorenz system (MCLS) [8] where $D_{KV} = 2.23542$. Consequently, the one-parameter attractor described in (3) is highly chaotic at $b \approx 296.5$, and is therefore particularly well suited for an application to a private communication system.

By using the Fourth-order Runge-Kutta method to solve the system (3) with time step size of 0.001, Figure 2 displays the resulting strange attractor of (3) using the initial values $x(0) = 1, y(0) = 1$ and $z(0) = 6$. It appears from Fig. 2 that the new attractor exhibits abundantly complex behaviour of chaotic dynamics.

![Figure 2: A one-parameter highly chaotic attractor, (a) x-y plane, (b) y-z plane, (c) x-z plane.](image)

3. Dual-Channel Single-Attractor Private Communications

An existing chaotic masking technique [5,6] can be modified using the one-parameter highly chaotic attractor described in (3), as a drive system at the transmitter and a response subsystem at the receiver, for a dual-channel single-attractor private communication system, as shown in Fig. 3.

3.1. A Dual-Channel Single-Attractor Transmitter

At the transmitter, the one-parameter highly chaotic attractor described in (3) can be used as a single drive system for a dual-channel transmitter independent of its response subsystem at the receiver as follows:
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As shown in Fig. 3, the dual-channel transmitter consists of two parallel transmitted signals. The first transmitted signal is \( s_1(t) = x_1(t) + m_1(t) \), where \( x_1(t) \) is a chaotic masking signal and \( m_1(t) \) represents the first original message to be transmitted. The second transmitted signal is \( s_2(t) = y_1(t) + m_2(t) \), where \( y_1(t) \) is a chaotic masking signal and \( m_2(t) \) represents the second original message to be transmitted. As suggested in [5], the power spectra of the chaotic masking signals should be much higher than those of the messages, i.e. they should be highly overlapping with an average signal-to-masking ratio of approximately \(-20\) dB.

3.2. A Dual-Channel Single-Attractor Receiver

At the receiver, the one-parameter highly chaotic attractor described in (3) can be used as a single response subsystem for a dual-channel receiver as follows:

\[
\begin{align*}
\dot{x}_r &= 10[y_r(t) - x_r], \\
y_r &= (296.5)x_1(t) - 40s_1(t)z_r, \\
z_r &= 10s_1(t)y_r - z_r.
\end{align*}
\]

As shown in Fig. 3, the dual-channel receiver consists of two parallel received signals \( s_1(t) \) and \( s_2(t) \), each of which regenerates a clean masking signal \( y_r(t) \) and \( x_r(t) \), respectively. When the receiver synchronizes with \( s_2(t) \), then \( x_r(t) \equiv x_2(t) \). As shown in Fig. 3, the message signal \( m_1(t) \) can be recovered as \( \hat{m}_1(t) = s_1(t) - x_r(t) = x_1(t) + m_1(t) - x_2(t) \approx m_1(t) \). Similarly, when the receiver synchronizes with \( s_1(t) \), then \( y_r(t) \equiv y_1(t) \). As shown in Fig. 3, the message signal \( m_2(t) \) can be recovered as \( \hat{m}_2(t) = s_2(t) - y_r(t) = y_1(t) + m_2(t) - y_2(t) \approx m_2(t) \).

3.3. Numerical Results

As simple examples, the first transmitted message \( m_1(t) = 0.2\sin(2\pi f_1 t) \), where the frequency \( f_1 = 1\) kHz, and the second transmitted message \( m_2(t) \)
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is a pulse-train rectangular waveform with an amplitude \( = 0.4 \) and the frequency \( f_2 = 2 \text{ kHz} \). Self-synchronization can be achieved over a wide range of initial conditions, e.g. at the transmitter \( [x_t(\omega), y_t(\omega), z_t(\omega)] = [2, 2, 6], \) and at the receiver \( [x_r(\omega), y_r(\omega), z_r(\omega)] = [-2, 4, 3]. \)

![Figure 4](image1.png)  
Figure 4: (a) A transmitted signal \( s_1(t) \), (b) A transmitted signal \( s_2(t) \).

![Figure 5](image2.png)  
Figure 5: (a) Recovered message \( m_1(t) \), (b) Recovered message \( m_2(t) \).

![Figure 6](image3.png)  
Figure 6: Synchronization of chaotic masking signals (a) \( x_r(t) \) versus \( x_t(t) \), (b) \( y_r(t) \) versus \( y_t(t) \).

Figures 4(a) and 4(b) show the transmitted signal \( s_1(t) \) and \( s_2(t) \), respectively. Figures 5(a) and 5(b) depict the recovered messages \( m_1(t) \) and \( m_2(t) \), respectively. Figure 6(a) displays the synchronization of the clean chaotic masking signals \( x_r(t) \) versus \( x_t(t) \) whilst Fig. 6(b) shows the synchronization of the clean chaotic masking signals \( y_r(t) \) versus \( y_t(t) \). Figure 7(a) illustrates a transient prior to the successful synchronization of...
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signals \( x_t(t) \) and \( x_r(t) \). Plots of errors \( |x_r - x_t| \), \( |y_r - y_t| \) and \( |z_r - z_t| \) are depicted in Figs 7(b), 7(c) and 7(d), respectively.

Figure 7: Transients of (a) signals \( x_t(t) \) and \( x_r(t) \), (b) errors \( |x_r - x_t| \), (c) errors \( |y_r - y_t| \) and (d) errors \( |z_r - z_t| \).

3.4. An Alternative Dual-Channel Single-Attractor Receiver

Figure 8: An alternative dual-channel single-attractor receiver based on the one-parameter highly chaotic attractor.

As an alternative to the receiver shown Fig. 3, Fig. 8 shows another possible dual-channel single-attractor receiver whilst the similar transmitter remains the same. The one-parameter highly chaotic attractor described in (3) can be used as a single response subsystem for an alternative dual-channel receiver as follows.
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\[
\begin{align*}
x'_r &= 10[s_2(t) - x_r], \\
y'_r &= (296.5)s_1(t) - 40x_r(t)x_r, \\
z'_r &= 10s_1(t)y_r - z_r.
\end{align*}
\]  
(6)

A major disadvantage of the system (6), however, is the required longer transient time prior to the steady synchronization.

4. Conclusions

A one-parameter highly chaotic attractor has been presented and its application to a dual-channel, single-attractor, private communication system has been demonstrated based on self-synchronization and chaotic masking techniques. Only a single attractor is required for a dual-channel transmitter or receiver, and can be either the well-known Lorenz attractor, the Lorenz-like attractor, or the one-parameter highly chaotic attractor developed in this paper. The latter is particularly well suited for an application to private communications due to the highest value of the maximum Lyapunov exponent of 2.6148 and the relatively high value of the maximum Kaplan-Yorke dimension of 2.1921. Twice increase in higher speed can be expected due to the dual channel.

References