Compound Structures of Six New Chaotic Attractors in a Modified Only-Single-Coefficient Jerk Model Based on Sinh⁻¹ Nonlinearity

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Abstract: Six new chaotic attractors in a modified only-single-coefficient jerk model are presented based on six new sets of a single coefficient and hyperbolic arcsine nonlinearity. In particular, compound structures of such chaotic attractors are demonstrated through the use of a half-image operation using a control parameter \( n \). The positive value of an appropriate \( n \) isolates the right half-image attractors whilst the negative value of an appropriate \( n \) isolates the left half-image attractor. Both images can then be merged together as a compound structure.

Keywords: Chaos, Jerk Model, Compound Structure, Single Coefficient, Hyperbolic Arcsine Nonlinearity.

1. Introduction

The well-known Chua’s chaotic oscillator [1,2] has been based on three first-order differential equations (ODEs). In contrast, Sprott [3] has alternatively proposed chaotic oscillators based on a single third-order ODE in a “jerk model” with an only single coefficient \( K \) of the form

\[
\frac{d^3x}{dt^3} + K \frac{d^2x}{dt^2} + \frac{dx}{dt} = G(x)
\]  

(1)

The term “jerk” comes from the fact that in a mechanical system in which \( x \) is the displacement, successive time derivatives of \( x \) are velocity, acceleration, and jerk [4]. The nonlinear component \( G(x) \) of such a single coefficient jerk model has been suggested as follows:

\[
G(x) = \begin{cases} 
|\chi| - 2 & ; K = 0.6 \ [3] \\
-6 \max(x,0) + 0.5 & ; K = 0.6 \ [3] \\
-4.5 \sgn(x) + 1.2x & ; K = 0.6 \ [3] \\
2 \sgn(x) - 1.2x & ; K = 0.6 \ [3] \\
2 \tanh(x) - x & ; K = 0.19 \ [5] \\
3 \sin(x) - x & ; K = 1 \ [6] \\
6 \tan^{-1}(x) - 2x & ; K = 1 \ [6] \\
7 \tanh(x) - 2x & ; K = 1 \ [6] \\
\sgn(x) - 2x & ; K = 1 \ [6] 
\end{cases}
\]  

(2)
Some of such jerk models have been implemented using current-feedback op-amps \cite{7,8}. In addition, new sets of the single coefficient $K$ have been presented based on new sets of either $\tan^{-1}$ nonlinearity \cite{9} or $\sin^{-1}$ nonlinearity \cite{10}. Recently, compound structures of chaotic attractors based on the single-coefficient jerk model \cite{9,10} and compound structures of others \cite{11,12,13} have been proposed.

In this paper, six new chaotic attractors in a modified only-single-coefficient jerk model are presented using six new sets of a single coefficient and hyperbolic arcsine nonlinearity. In addition, compound structures of each chaotic attractor are also demonstrated.

2. Modified Only-Single-Coefficient Jerk Model

2.1. Six New Sets of Coefficients and Nonlinearity

Figure 1 shows a structural implementation of the jerk model described in (1) where the only single coefficient $K$ and the nonlinearity $G(x)$ can now be modified. By varying the coefficient $K$ and the nonlinear component $G(x)$ through computer simulations, six new sets of the coefficient $K$ with new nonlinearity based on hyperbolic arcsine functions are proposed as follows:

\begin{equation}
G(x) = \begin{cases} 
G_1(x) = +4\sinh^{-1}(x) - x; K = 0.24 \\
G_2(x) = +5\sinh^{-1}(x) - x; K = 0.26 \\
G_3(x) = +6\sinh^{-1}(x) - x; K = 0.32 \\
G_4(x) = -4\sinh^{-1}(x) + x; K = 0.19 \\
G_5(x) = -5\sinh^{-1}(x) + x; K = 0.21 \\
G_6(x) = -6\sinh^{-1}(x) + x; K = 0.23
\end{cases}
\end{equation}

Figure 1. An only-Single-Coefficient Jerk Model.
1. Compound Structures of New Chaotic Attractors

In the new systems (1) and (3), compound structures [9-13] may be demonstrated using a half-image operation to obtain only the left or the right half-image attractors, both of which can then be merged together as a compound structure. Such an operation can be revealed through the use of a controlled system of the form:

\[
\frac{d^3 x}{dt^3} + K \frac{d^2 x}{dt^2} + \frac{dx}{dt} = G(x) + n
\]

where \(n\) is a control parameter. A negative value of an appropriate \(n\) results in an isolation of the left-half image of the original attractor. On the contrary, a positive value of an appropriate \(n\) results in an isolation of the right-half image of the original attractor.

2. Numerical Results

3.1. New Chaotic Attractors

By using the only-single-coefficient jerk model described in (1) with six new sets of \(K\) and \(G_1(x)\) to \(G_6(x)\) described in (3), six new chaotic attractors are displayed, respectively, in Figs. 2-(A1, B1, C1, D1, E1 and F1) for the X-Y phase plane, and in Figs. 2-(A2, B2, C2, D2, E2 and F2) for the X-Z phase plane. It appears that the new attractors exhibit complex behaviors of chaotic dynamics.

3.2. Compound Structures of New Chaotic Attractors

For the nonlinearity \(G_1(x)\) at \(n = -0.09\), a left-half image of its original attractor [see Fig. 2-(A1 and A2)] can be isolated as illustrated in Figs. 2-(A3 and A4). In contrast, at \(n = 0.09\), another right-half image of its original attractor can be isolated as illustrated in Figs. 2-(A5 and A6).

For the nonlinearity \(G_2(x)\) at \(n = -0.45\), a left-half image of its original attractor [see Fig. 2-(B1 and B2)] can be isolated as illustrated in Figs. 2-(B3 and B4). In contrast, at \(n = 0.45\), another right-half image of its original attractor can be isolated as illustrated in Figs. 2-(B5 and B6).

For the nonlinearity \(G_3(x)\) at \(n = -0.78\), a left-half image of its original attractor [see Fig. 2-(C1 and C2)] can be isolated as illustrated in Figs. 2-(C3 and C4). In contrast, at \(n = 0.78\), another right-half image of its original attractor can be isolated as illustrated in Figs. 2-(C5 and C6).

For the nonlinearity \(G_4(x)\) at \(n = -0.15\), a left-half image of its original attractor [see Fig. 2-(D1 and D2)] can be isolated as illustrated in Figs. 2-(D3 and D4). In contrast, at \(n = 0.15\), another right-half image of its original attractor can be isolated as illustrated in Figs. 2-(D5 and D6).
For the nonlinearity $G_1(x)$ at $n = -0.21$, a left-half image of its original attractor [see Fig. 2-(E1 and E2)] can be isolated as illustrated in Figs. 2-(E3 and E4). In contrast, at $n = 0.21$, another right-half image of its original attractor can be isolated as illustrated in Figs. 2-(E5 and E6).

Finally, for the nonlinearity $G_6(x)$ at $n = -0.29$, a left-half image of its original attractor [see Fig. 2-(F1 and F2)] can be isolated as illustrated in Figs. 2-(F3 and F4). In contrast, at $n = 0.30$, another right-half image of its original attractor can be isolated as illustrated in Figs. 2-(F5 and F6).

<table>
<thead>
<tr>
<th>New Original Attractors</th>
<th>Left half-image Attractors $n = -0.09$</th>
<th>Right half-image Attractors $n = 0.09$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1(x) = 4\sinh^{-1}(x) - x$ ($K=0.24$)</td>
<td><img src="image1.png" alt="A1" /></td>
<td><img src="image2.png" alt="A3" /></td>
</tr>
<tr>
<td>$G_2(x) = 5\sinh^{-1}(x) - x$ ($K=0.26$)</td>
<td><img src="image3.png" alt="A2" /></td>
<td><img src="image4.png" alt="A4" /></td>
</tr>
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<td></td>
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<td><img src="image6.png" alt="A6" /></td>
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<td><img src="image7.png" alt="B1" /></td>
<td><img src="image8.png" alt="B3" /></td>
</tr>
<tr>
<td>$G_4(x) = 6\sinh^{-1}(x) - x$ ($K=0.26$)</td>
<td><img src="image9.png" alt="B2" /></td>
<td><img src="image10.png" alt="B4" /></td>
</tr>
<tr>
<td></td>
<td><img src="image11.png" alt="B5" /></td>
<td><img src="image12.png" alt="B6" /></td>
</tr>
</tbody>
</table>

Figure 2. Six new chaotic attractors and their compound structures isolated into left and right half-image attractors.
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\[
G_3(x) = 6\sinh^{-1}(x) - x \quad (K=0.32)
\]

\[
G_4(x) = -4\sinh^{-1}(x) + x \quad (K=0.19)
\]

\[
\begin{align*}
\text{C1} & : n = -0.78 \\
\text{C3} & : n = -0.15 \\
\text{C5} & : n = 0.78 \\
\text{C2} & : n = 0.15 \\
\text{C4} & : n = 0.78 \\
\text{C6} & : n = 0.15
\end{align*}
\]

Figure 2. Six new chaotic attractors and their compound structures isolated into left and right half-image attractors (continued).
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\[ G_5(x) = -5 \sinh^{-1}(x) + x \]  
(K=0.21)  
(E1)

\[ n = -0.21 \]  
(E3)

\[ n = 0.21 \]  
(E5)

\[ G_6(x) = -6 \sinh^{-1}(x) + x \]  
(K=0.23)  
(F1)

\[ n = -0.29 \]  
(F3)

\[ n = 0.30 \]  
(F5)

\[ G_7(x) = -6 \sinh^{-1}(x) + x \]  
(K=0.25)  
(F2)

\[ n = -0.30 \]  
(F4)

\[ n = 0.30 \]  
(F6)

Figure 2. Six new chaotic attractors and their compound structures isolated into left and right half-image attractors (continued).

5. Conclusions

Six new chaotic attractors in a modified only-single-coefficient jerk model have been proposed using six new sets of a single coefficients and hyperbolic arcsine nonlinear function. In addition, compound structures of all six chaotic attractors have been demonstrated using a half-image operation to obtain only the left or the right half-image attractors, both of which can be merged together as a compound structure.
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References


