Mathematical simulation of urbanization processes based on analogies with physical fractals

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Abstract: Proposed attributive and structural analogy between physical fractal and social economic system. Proposed algorithm for growth modeling of social economic system. Researched mechanism of stagnation appearance and system self-organization in evolution process. Specified functional analogy between crystal entropy and average level of system competition.

Keywords: fractal, attractor, self organization, entropy, phase transition.

1. Introduction
Possibility of simulation and manage development of different types of the difficult socio-economic systems is the important element of the effective decision-making on different levels. Such systems may include development of different forms of tourism, infrastructures, development of resort settlements, informative streams etc.
Region development would take place more dynamically if the program of development was based on scientific results, got by quantitative methods which enable to solve concrete tasks with the construction of forecasting scenarios and possibility to foresee formation and development of difficult socio-economic processes. Nowadays an effective mathematical tool is developed which allows simulating development of the difficult physical systems such as fractal growth of crystals, simulation of diffusion and transfer of energy, crystallization, superficial effects, area and optical spectrums.
A research purpose was a simulation of development difficult socio-economic systems which based on analogies between the phenomena of fractal growth of crystals, with further establishment of functional analogies. Research actuality consists of development methods for prognostication the structure of difficult social processes based on fractal growth of crystals and molecular dynamics methods, and also methods of fuzzy logic. To such processes it is possible to take functional reorganization of cities and settlements, which are related to active development of different forms of tourism, adaptation of concomitant infrastructure etc.
Geometry of settlements growth prognostication and their underlying structure will allow planning reorganization of the proper infrastructure and
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communications with a maximal economic value. There is possibility to foresee the structure of new buildings in the vicinity of the accrued tourist recreational systems (TRS). It, in same queue, will allow optimizing strategy of reorganization of TRS, define specialization for separate segments of settlement and foresee the money streams of such system [1].

2. Methodology

Development of TRS is conditioned by a presence and development of centers of attractiveness for recreation. A mountain-skier lift, healthful source, historical-cultural object etc, can exemplify such centers of attractiveness centers for recreation. With the development of these centers gradually the local population joins which adopts on itself number of service functions: hotel and motel service, rent, food and other. As a rule, those, whose business is located near the center of attractiveness, develop most actively. In this connection, it is possible to establish that urbanization of territory will broaden from a center, and will be directed along the roads. As transport communication is a basic temporal factor for transferring to the center of TRS from the place of receiving service services. It is possible to mention that a similar mechanism in the physical systems results to the dendritic growth, for example crystals.

Development of GPS technologies (Global Positioning System), various international programs of remote sensing, creation of digital maps are open new possibilities for research-and-development scientific approaches in geoinformatics, architecture and other spheres of sciences. In particular it is possible to mention that snapshots of the urban systems on macrolevel (settlements and elements of infrastructure) indeed have many general lines with physical fractals (Fig. 1), namely: clearly expressed centers of origin,
dendritic growth and presence of well-organized structure in the central area. According to determination of Mandelbrot: physical fractal is a geometric object (line, surface, body), having the strongly cut up structure and property of self similarity in the limited scale [2]. The mathematical tool for simulation the growth of physical fractals is well investigated. Therefore we will examine a physical fractal (A) as a basis (analogue) for the design of the socio-economic system (B). For subsequent research on the basis of analogies, we will select own properties (attributive analogy) for objects A and B which are always inherent an object and not depend on situations or processes it takes part in [3]. It is possible to take to them: a fuse (attractor) is a center of origin of physical crystal. We can exemplify fuse as a defect, foreign body, fluctuation etc. Attractors for the socio-economic systems are historical-cultural legacy, mountain-skier lifts, entertaining centers, beaches, manufactures etc. Dendritic growth in crystals is conditioned by free atoms (by molecules) which aggregate on the condensed cluster during Brownian motion. New socio-economic elements appear in a direct closeness from an attractor or neighbors. The deformation of crystals is caused by the anisotropy of environment which can be described by the potential field. The field of attractiveness which depends on distance, infrastructure, relief, legislation and other aspects, plays the role of the potential field in social processes. A change of internal symmetry of physical fractals is caused spontaneous evaporation of atoms which have weak intermolecular interactions. Disappearance of the socio-economic system elements related to the competition. By other words, objects which have a high level of competition and low competitiveness disappear. Inertness: in physics is mass \( m \) measure the inertness of solid. Thus, the bigger is the mass of particle, the less is the influence of the potential field on its motion. The reducing of mass leads to the increasing of acceleration towards the maximal gradient of the potential field. In the socio-economic systems the potential field of attractiveness is maximal in the neighborhood of existent infrastructure. Elements with the low level of investment possibility will gravitate to the existent infrastructure (for example to the road). Elements of large business, as a rule, either create an infrastructure or localized in a direct closeness from attractors. Therefore mass can be interpreted as a measure of investment ability of certain object of settlement (sanatorium, hotel, office, cottage, summer residence etc.) or infrastructure (supermarket, shop, booth etc.). Medium viscosity: diminishes kinetic energy of free particles and hindering motion toward the maximum of the potential field gradient. The closeness of environment \( \beta \) can be interpreted as a size reverse to the investment assistance of a region. The more viscosity the less formation assistance of new elements in the attractor neighborhood (Tab. 1.).

Analogues existence of own properties and likeness of objects between itself (Fig. 1) allow to suggest a hypothesis about existence of structural analogy between them. Let us set \( S_i(\xi) \) – internal communication...
Then a structural analogy is determined as: \( \exists S_j(A), \exists S_j(B), \{S_j(A) \approx S_j(B)\} \). If this hypothesis is true then between objects there must be a functional analogy. Let us set \( F_i(\xi) \) – function \( i \) of object \( \xi \). Existence of functional analogy foresees implementation of condition: \( \exists F_i(A), \exists F_j(B), \{F_i(A) \approx F_j(B)\} \).

For proof of functional analogies it is needed to model socio-economic system growth using methods of fractal growth of crystals and research functional connection of the fractal.

Growth of settlements simulation algorithm by the methods of fractal growth of crystals developed by an author in detail described in study [4]. The block scheme of simulation algorithm is presented on Fig. 2.

### 3. Results

For model approbation was conducted an experiment which
simulates the fractal structure of settlements: Vorokhta of the Ivano-Frankivsk region and Budilov of the Chernivtsy region.

The choice of mentioned settlements as model approbation is explained that Vorokhta has urban-type settlement status and characterized by a difficult structure (has a few access roads, mountain-skier lifts and tourist centers). Budilov, unlike Vorokhta, is a small village where buildings are located along roads, there are a few access roads and railway station.

Input parameters of the model were: Vorokhta – attractors: coordinates of mountain-skier lifts, springboards, center of the city; vectors of access roads. In the case of Budilov – attractors: center of the village; vectors of roads and streets; railway. For forming production rules of fuzzy model was used landscape information of the urbanized territories of Carpathians region, got using GIS. Relief features were not taken into account.

Calculations were carried out by the modified diffusely limited aggregation (DLA) method with such approaching: initial speed of particle got out randomly; mirror maximum terms were used [5]. That is, crossing the verge of the probed area, a particle appeared from an opposite side, saving all other dynamic indexes here. During the solving optimization task for Vorokhta the best result is got at $m \approx 0.5, \beta \approx 10^{-4}, d = 1$.

Fig. 3 represents a prognosis fractal structure of Vorokhta in obedience to the offered methodology which consists of about 21 000 aggregate particles compared to the picture of this locality from space. A ratio error of the forecast urbanized area from the real was 6%.

Simulating fractal structure of Budilov the best agreement with GIS was got at $m \approx 0.1, \beta \approx 10^{-3}, d = 0.01$ (Fig. 4). An error was 8% in this
case. From a picture evidently, that the presence of railway does not influence as on the form of fractal so on the form of settlement. From the found values of mass, the closeness of environment and coefficient of evaporation we can draw a conclusion that Budilov is at the beginning of forming, because fractal is formed by easy particles which are responsible for small money streams. The particles of greater mass which testifies to the large money and investment processes take part in forming of Vorokhta. It is also confirmed by the less resistance of environment of Vorokhta that prove more favorable atmosphere to development. There are processes of evaporation in Vorokhta, unlike Budilov where they almost are absent. We can see competitive activity in Vorokhta during which new elements appear and other disappear.

It is known that the chaotic change of inner structure in many cases can lead to regulation of the system. In thermodynamics the measure of «order of the system» is entropy. A computer model allows defining the state of the system at any moment of time that is why entropy defines by the Boltzmann formula [6]:

\[
S = k \ln(\Omega),
\]

where \( k \) – Boltzmann constant, \( \Omega \) – is a number of the microstates that are possible in this macrostate of the system.

For the calculation of entropy fractal was broken up on elementary cells and then analysis of their possible states was conducted. State will define as the amount of neighbors of particle and their spatial configuration.

Fractal grew in an isotropic environment on one fuse. Fig. 5 shows the dependence chart of entropy for \( k = 0,85 \) compared to the change of

![Fig. 4. Budilov](image)

a) picture from space, b) forecasting fractal structure
middle level of the system competition. From a picture is evident that both curves have an extreme point after which there is an asymptotic slump. Positions of maxima of curves and asymptote well correlate between themselves. Diminishing of entropy is explained that the system is thermodynamically opened. Consequently, it is possible to establish the presence of functional analogy between the middle level of competition and entropy of the system.

On a Fig. 6 there are dependences of amount of particles on their level of competition. This curves can be approximated by Maxwell distribution. In thermodynamics distributing of particles after internal or kinetic energy and other dynamic indexes is also described by Maxwell dependence. Consequently, it is possible to assume the presence of functional dependence between kinetic energy in a crystal and level of competition in the socio-economic system. If assumption is faithful, then thermodynamics entropy must coincide with kinetic energy in the Boltzmann formula. For the calculation of thermodynamics entropy it is necessary to define internal energy of the particle:

\[ Q_i = E_i + U_i, \]

where \( E_i \) – is kinetic energy, \( U_i \) – is potential energy.

Then thermodynamics entropy:

\[ \Delta S = \sum \frac{\Delta Q_i}{T}, \]

where \( T \) – is a temperature of the system.

As calculations show, thermodynamics entropy is practically identical to the middle level of competition (Fig. 6). That confirms a functional analogy between kinetic energy of particle and middle level of competition of the system.

Most settlements have a transport connection. Therefore more reliable approaching is a model of settlement of the attractor and which is crossed by a road. Results of calculation for the entropy are changed by the
Boltzmann formula in this approaching and are presented on Fig. 7. As we see from a picture, on graphic are presented two clearly expressed peaks. Appearance of the second peak is conditioned by a change of internal symmetry of the crystal and it is the indicator of second type phase transition. Indeed, in small settlements home are disposed along roads and streets are parallel to the road. That symmetry is one-dimensional. This structure is usual for most of settlements. Organization after this rule is corresponding to the first peak (Fig. 7.b.). In future the settlement begins to grow deeply from a road. That’s why symmetry of crystal changes which becomes two dimensional. Perpendicular and radial streets appear (Fig. 7.c.). Such structures are usual for cities. Symmetry change causes appearance of the second peak.

Fig. 8 represents the charts of entropy change for the fractals of Budilov and Vorokhta. For analysis comfort a maximal value on y-axis corresponds the value of entropy for fractals presented on Fig. 3 and 4. Beginning of counting out coincides with the moment of fractal birth. From a picture is evident, that Vorokhta is in a state of the second phase transition. It means that fractal, which simulate this settlement passed in the development of a village structure and changes it on a city structure. Therefore Vorokhta on this stage has features inherent the city in the neighborhood of center and suburbs have one-dimensional symmetry. That is indeed observed in practice. A fractal of Budilov is on the initial stage of forming and did not yet go out on the extreme point of entropy. That confirms the fact of a few configurations that inherent unidimensional symmetry. Therefore Budilov is a classic small village.
4. Conclusions

In this study an attributive and structural analogy between physical fractals and socio-economic systems were set. Methodology of simulation the socio-economic systems development by methods of fuzzy logic, fractal growth of crystals and molecular dynamics was formulated. Author has offered method for evaporation simulation in the process of crystal growth which allowed explaining the basic phenomena that are observed in the real socio-economic systems. In particular, in the process of evolution there is a point of stagnation after which the level of general competition of the open system diminishes. It confirms the fact of self-organization of the system.

A functional analogy is set between entropy of crystal and middle level competition of the socio-economic system and also between kinetic energy and level of competition. Appearance of second type phase transitions were studied which determine symmetry changes of settlements, and can serve as the indicators for the state of the socio-economic system. The offered methodology was approved on town Vorokhta and village Budilov. We got fractals which well describe both external and internal descriptions of the real objects that confirm adequacy of methodology.

Fig.8. Entropy changing for Budilov fractal (1) and Vorokhta (2) depending on algorithm iteration.
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