Von Karman Streets Chaotic Simulation

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In this paper we model and simulate the so-called Von Karman Streets, a characteristic form of turbulent (vortex) flow that appears in small or large systems. Very interesting examples are the large turbulent formations in the sea or in the clouds that have been viewed and photographed from the space (from satellites). The modeling approach is based on the Rotation-Reflection-Translation theory developed in the recent book [1]. The related theory was already applied in various fields and especially in flows.

Keywords: Von Karman Streets; Chaotic Modeling; Chaotic Simulation; Vortex Flow; Difference Equations; Rotation-Translation; Reflection-Translation.

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1. Introduction

The famous Von Karman chaotic phenomena are presented in several formations, especially in the sea and are illustrated by photographs taken from space (satellite views).

The Von Karman phenomenon appears when a fluid flowing past an obstacle that appears in the form of a cylinder (A vortex street develops behind a cylinder moving through fluid) or an island in the ocean. In the later case the flow (fluid) lines or trajectories pass from both sides of the island. To simplify the modeling we observe that a mirror image of the flow appears in the other side of the island or a reflection like process appears.

2. Chaotic Modeling

The chaotic theory from its first application of the famous Lorenz[2] model gave rise to a new modeling approach that is the search for the main characteristics of the phenomenon thus neglecting the secondary characteristics. This was applied by Lorenz who showed that a fluid flow model could be modeled by only 3 basic equations instead of the numerous and more complicated equations that where suggested from the fluid flow theory and

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the Navier-Stokes equations. Summarising the related achievements of several chaotic modeling applications we may observe that the modeling of a chaotic system could result in good approach to the real situations if:

a) The main chaotic-nonlinear characteristic part of the process is retained
b) The number of equations expressing the phenomenon is reduced to a minimum.
c) The "chaotic parameters" selected clearly represent the main nonlinear characteristics of the system.

Another modeling issue is the selection of the form of the chaotic system of equations by means of the selection between a set of differential and a set of difference equations. The differential equation theory is much more studied in details the last centuries instead of the difference theory approach.

However, the difference theory approach looks more essential to model chaotic systems especially from the simulation point of view.[1,3–5] Usually to simulate a set of difference equations we need only a simple "loop" in a computer program to model the iterative process. Instead the differential equations simulation needs more laborious methods.

In our case of modeling the Von Karman Streets we observe that the flow is around a solid form like a cylinder represented by an island in the ocean (The Island is assumed to have a circular form). We assume that a fluid (air or water) flow is directed from left to the right of the coordinates, the island located at the centre of coordinates at $(0, 0)$. The flow (fluid) lines or trajectories past both sides of the island. To simplify the modeling we observe that a mirror image of the flow appears in the other side of the island or a reflection like process appears provided that a symmetric island form is present (In real situations there is not a perfect symmetry present, but a non-perfect mirror image may also provide a chaotic process). To model the phenomenon in the (x, y) plane we observe that:

(1) A fluid is flowing from left to the right with a speed represented by a parameter a. This parameter in geometric notation is noted as the "translation" parameter. The translation parameter is added to keep the direction of the flow. In modeling the process the direction of flow is parallel to the x axis.
(2) When the fluid lines past the island move following the circular form of the obstacle "rotating". Behind the island the rotating lines are joint together having opposite rotation directions thus forming a "reflection" phenomenon. We thus need a rotation equation set.

(3) In some instances the space contraction or expansion is present thus modeled by a space contraction parameter b.

The phenomenon can be modeled by using a reflection-translation equation set that we have introduced in [1]. The system has the form:

$$\begin{aligned} x_{t+1} &= a + b(x_t cos\theta_t + y_t sin\theta_t) \\ y_{t+1} &= b(x_t sin\theta_t - y_t cos\theta_t) \end{aligned} \tag{1}$$

where a is the translation parameter and b is the space contraction ($b < 1$) or expansion ($b > 1$) parameter. The rotation angle selected is of the form: $\theta_t = d/r^k$, d, k are parameters and $k > 0$ and $r = \sqrt{x^2 + y^2}$.

The rotation angle for the reflection-translation scheme must assure that the influence of the obstacle (island) is negligible in long distances, thus r should be in the denominator and the parameter k should be large. The parameter d accounts for the reflection and can be taken as 2π or π.

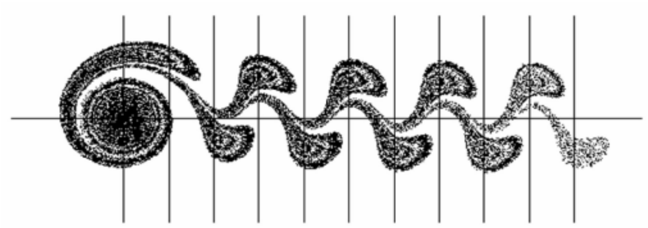

Fig. 1. Von Karman streets simulation (Parameters: $b = 1, d = 2\pi, a = 1, k = 3, t = 10$)

3. Chaotic Simulation

The simulation starts from an original circle of rotating particles with $R = 1$ at $(0, 0)$. The parameters selected are $b = 1, d = 2\pi, a = 1, k = 3, t = 10$. The resulting figure after $t = n = 10 iterations$ is illustrated in figure 1.

Behind the island the flow follows by forming chaotic forms or vortices. The distance between vortices (the frequency of vortex shedding) is equal to a.

Another point is to simulate the reflection-translation case but following a space contraction by means of a space contraction parameter < 1. For the simulation the parameter values of the previous case are retained except for the parameter $k = 4$ and for the space contraction parameter ($b = 0.9$). The resulting form is illustrated in Figure 2. The resulting chaotic forms slowly disappear under the space contraction process.

Fig. 2. Von Karman streets simulation (Parameters: $b = 0.9, d = 2\pi, a = 1, k = 4, t = 10$)

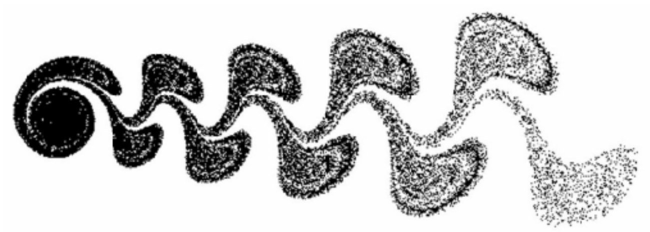

Fig. 3. Von Karman streets simulation (Parameters: $b = 1.1, d = 2\pi, a = 1, k = 3, t = 10$)

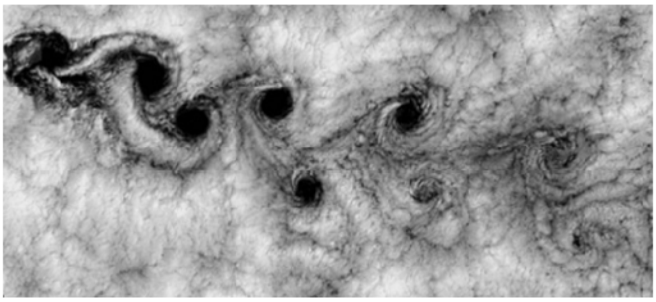

Fig. 4. Von Karman Vortex Street off the Chilean coast near the Juan Fernandez Islands.

The space expansion process ($b > 1$) is illustrated in Figure 3. The parameter values selected are as in the first case except for the space expansion parameter that is $b = 1.1$.

An example that follows the later space expansion case is illustrated in Figure 4. This is Landsat 7 image of clouds off the Chilean coast near the Juan Fernandez Islands (also known as the Robinson Crusoe Islands) on September 15, 1999. It is one of the Classical Von Karman Vortex Streets pictures.

(For more details on von Karman vortices, refer to http://climate.gsfc.nasa.gov/ cahalan, Image and caption courtesy Bob Cahalan, NASA GSFC).

According to Cahalan:[6]

> The island is about 1.5 km in diameter, and rises 1.6 km into a layer of marine stratocumulus clouds. This type of cloud is important for its strong cooling of the Earth's surface, partially counteracting the Greenhouse warming. An extended, steady equatorward wind creates vortices with clockwise flow off the eastern edge and counterclockwise flow off the western edge of the island. The vortices grow as they advect hundreds of kilometers downwind, making a street 10,000 times longer than those made in the laboratory.

The simulation is quite representative of the phenomenon, although it is only a 2-dimentional modeling of a 3-dimentional case. The introduction of a third equation expressing the vertical movements to the z direction would improve the simulation results.

4. Conclusions

In this paper we model and simulate the so-called Von Karman Streets by using difference equations approach modelling. The modeling approach based on the Rotation-Reflection-Translation theory was applied in the so-called Von Karman vortex streets with very promising results.

References

1. C. H. Skiadas and C. Skiadas, *Chaotic Modelling and Simulation: Analysis of Chaotic Models, Attractors and Forms* (Taylor & Francis / CRC Press, New York, 2008).
2. E. N. Lorenz, *J. Atmos. Sci.* **20**, 130 (1963).
3. C. H. Skiadas, *ArXiv.org* **arXiv:nlin/0611036v1**, 1 (2006).
4. C. H. Skiadas and C. Skiadas, *ArXiv.org* **arXiv:nlin/0701012v1**, 1 (2007).
5. C. H. Skiadas, Exploring and simulating chaotic advection: A difference equations approach, in *Recent Advances in Stochastic Modeling and Data Analysis*, ed. C. Skiadas (World Scientific, Singapore, 2007).
6. B. Cahalan, *NASA GSFC* `http://climate.gsfc.nasa.gov/cahalan` (1999).